Intro. to ODEs

Quiz 7 **Solutions**

1) Compute the determinants of the following square matrices.

a) $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & -1 \\ 2 & -1 & 4 \end{bmatrix}$

$$\begin{vmatrix} 1 & 2 & 3 \\ 1 & 0 & -1 \\ 2 & -1 & 4 \end{vmatrix} = -\begin{vmatrix} 1 & 0 & -1 \\ 1 & 2 & 3 \\ 2 & -1 & 4 \end{vmatrix}$$

$$= -\begin{vmatrix} 2 & 3 \\ -1 & 4 \end{vmatrix} + \begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix}$$

$$= -(8+3) + (-1-4) = -11 - 5 = -16$$

b) $B = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 1 & 1 & 1 & 1 \\ 2 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

$$\begin{vmatrix} 1 & 0 & -1 & 0 \\ 1 & 1 & 1 & 1 \\ 2 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{vmatrix} = -\begin{vmatrix} 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 2 & 0 & 1 & 1 \\ 1 & 0 & -1 & 0 \end{vmatrix}$$

$$= -\begin{vmatrix} 1 & 1 & 1 \\ 2 & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ 2 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} = -1$$

2) Find the values of λ that make the following determinant equal to 0.

$$\begin{vmatrix} 1-\lambda & 1 \\ 1 & -1-\lambda \end{vmatrix}$$

$$\begin{aligned} \begin{vmatrix} 1-\lambda & 1 \\ 1 & -1-\lambda \end{vmatrix} &= (1-\lambda)(-1-\lambda) - 1 \\ &= \lambda^2 - 2 = 0 \\ \lambda &= \pm\sqrt{2} \end{aligned}$$

3) Use row reduction to determine whether the following set of vectors are linearly independent. If they are dependent, find the ones that are independent and express the dependent ones as a linear combination of those.

$$\left\{ \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \\ -4 \end{bmatrix} \right\}$$

We begin by row reducing the matrix whose columns are the given vectors.

$$\begin{bmatrix} 1 & 2 & 1 & 1 \\ -1 & 0 & 1 & -1 \\ 1 & 1 & -1 & 0 \\ -1 & 1 & -1 & -4 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since there is a row of all zeros, the vectors are not linearly independent. Since there are three pivot columns, we know the first three vectors are linearly independent and the fourth can be written as a linear combination of them. The row reduction shows us that there are an infinite number of solutions to

$$c_1 \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} + c_2 \begin{bmatrix} 2 \\ 0 \\ 1 \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix} + c_4 \begin{bmatrix} 1 \\ -1 \\ 0 \\ -4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

given by $c_1 = -2c_4,\ c_2 = c_4,\ c_3 = -c_4$ with c_4 free. Taking $c_4 = -1$ gives

$$\begin{bmatrix} 1 \\ -1 \\ 0 \\ -4 \end{bmatrix} = 2 \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} - 1 \begin{bmatrix} 2 \\ 0 \\ 1 \\ 1 \end{bmatrix} + 1 \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}.$$